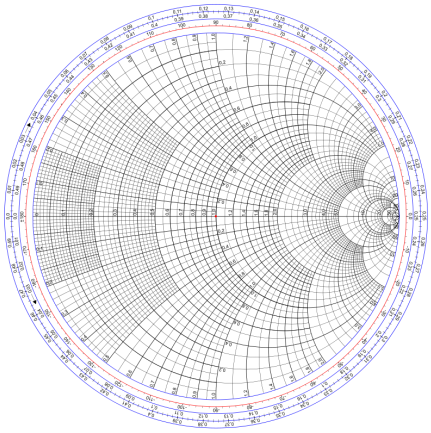
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An in-depth look at a radio-related topic



The Smith chart

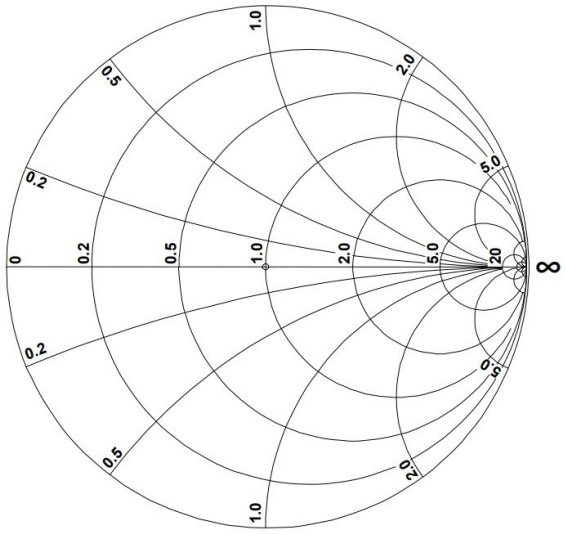


Very few new amateurs have ever heard of it. Modern RF (radio frequency) engineers cringe when they see one. Those who pass the Extra exam have likely encountered it while studying, but still have little idea what it's used for. In fact, most hams live their entire amateur lives having never seen, nor even understood the relevance of a ***Smith chart*** to amateur radio, and that's alright. But for those who are truly curious, or really want to know, this is your chance. I've added the corresponding Extra exam questions in parentheses.

A Smith chart is a graphical tool for solving engineering problems with transmission lines and matching circuits *at one particular frequency*, and can be used to calculate impedance (E9GØ1) and SWR (E9GØ3) along a transmission line, for example. After Phillip Smith learned how to use a slide rule, he became fascinated with graphical representations of mathematical relationships. He and two others applied a polar diagram to the complex plane, and they published its forerunner, the *transmission line chart* in 1939. Technically, it displays a mathematical transformation (map) of the two-dimensional Cartesian complex plane.

Construction

To better understand how a Smith chart works, let's examine how it's constructed. It's basically made of two sets of lines in the shapes of circles and arcs. All the lines on the chart share the right end as a common point. The arcs are drawn from that right-end point and orthogonal (microscopically perpendicular) to the circles, and are circle sections themselves. Impedance **Z** is described by the complex number **R + jX**, the real portion **R** being resistance and the imaginary portion **X** being the reactance. Applying this, a chart circle represents the resistance while an arc represents the reactance (E9GØ2, E9GØ4) of any impedance value. This means all points along a particular chart circle indicate the same resistance while all points along a particular chart arc indicate the same reactance (E9G1Ø).

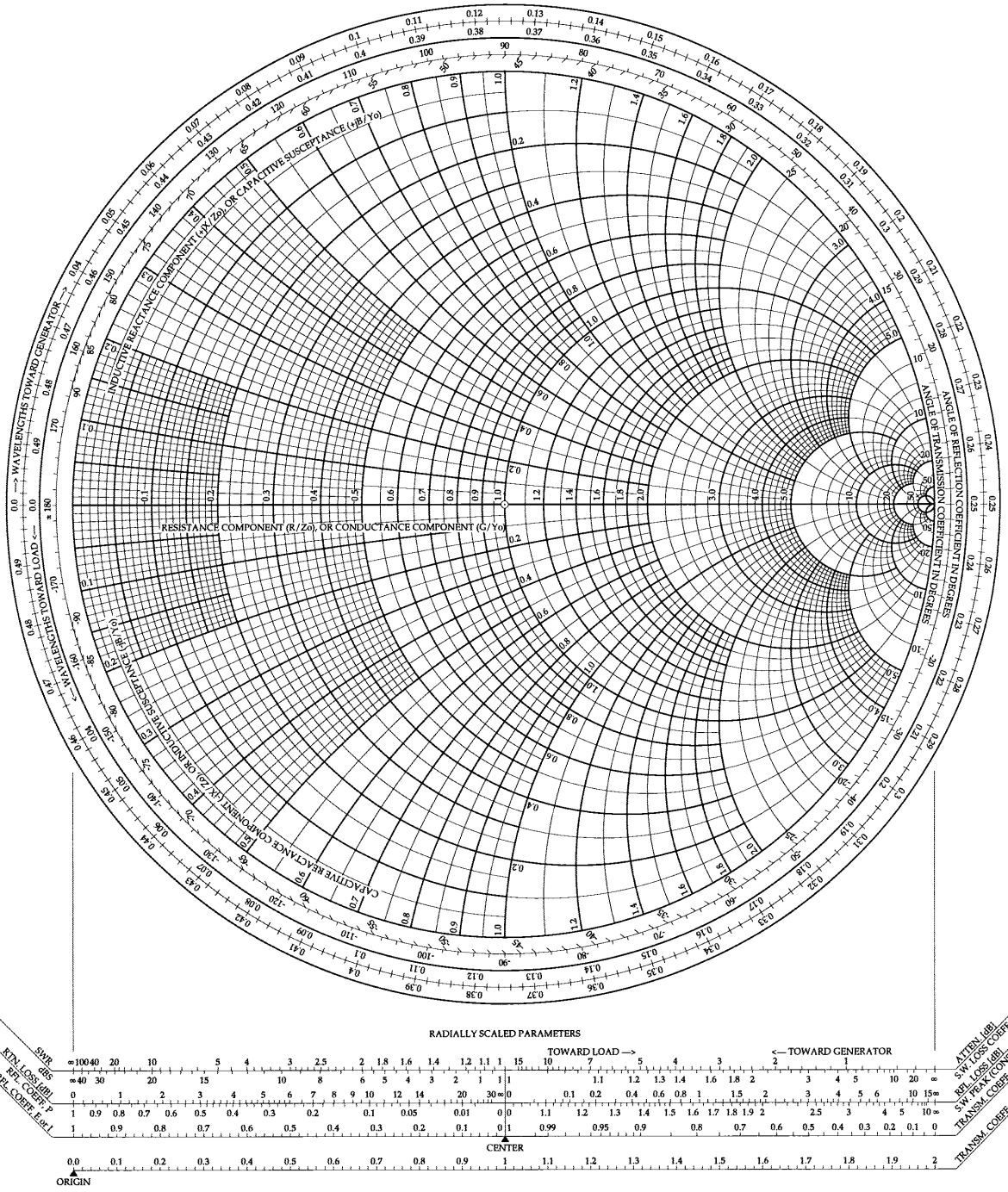


All arcs terminate at the large outside circle, which represents the reactance axis (E9GØ6). The horizontal line through the middle is the only straight line and represents the resistance axis (E9GØ7). The "origin" (also known as the *Prime Center*) of the Smith chart is the point at which **Z = 1.Ø + jØ.Ø**, or a resistance of **1.0** and a reactance of **Ø.Ø**. The left end is where **Z = 0.0 + j0.0** and the right end is where **Z = ∞ + j0.0**. At first glance, the chart appears somewhat logarithmic, but in fact it's got unique graduations.

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Sample Smith chart, complete with horizontal rule

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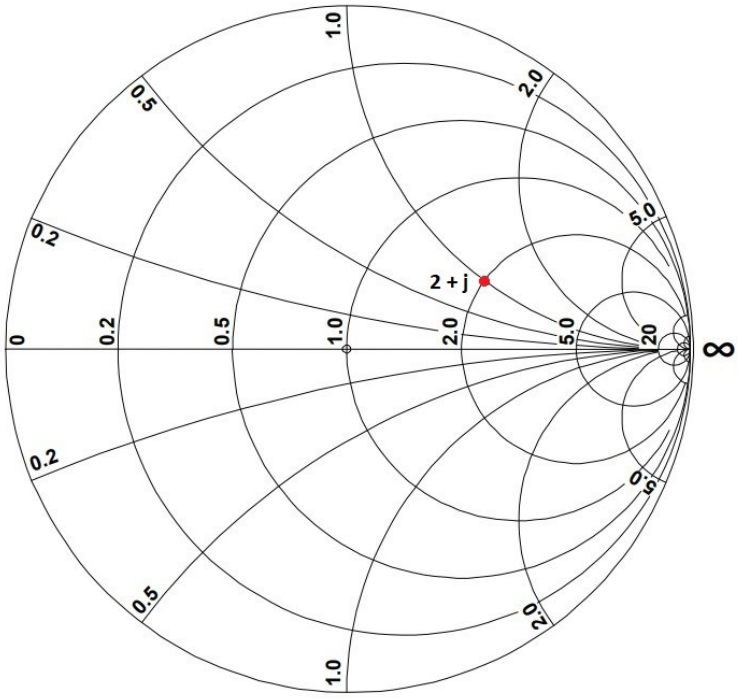
The circular shape of the Smith chart is representative of half of a wavelength distance along a transmission line. Only one half-wavelength is needed, since the mathematical transformation of the second half is a repeat of the first. Around the outside of the reactance axis is a set of (at least) three numbers. One represents the angular "distance" from 0° to 180°, and the other from 0° to -180°. The other two are fractions of a wavelength from 0 to 0.5 counter-clockwise (toward the load) and 0 to 0.5 clockwise (toward the generator, which is the transmitter, because the transmitter *generates* the signal) (E9G11).

Plotting a point

As an example, let's plot a load (antenna) impedance of **100 + j50** ohms. Because the Smith chart can only plot values that are visually meaningful to the reader, it's customary to ***normalize*** the complex values by dividing by the characteristic impedance (E9GØ8). In most amateur radio cases, that value will be **50 ohms**, so normalizing our example term gives us

$$\frac{100 + j50\ \Omega}{50\ \Omega} = 2 + j$$

The **2 + j** value indicates where the **+2.0** resistance circle intersects with the **+1.0** reactance arc and results in the following plot (red dot):



Note that resistances are plotted only in the positive direction, because it's assumed we'll never be dealing with negative resistances.

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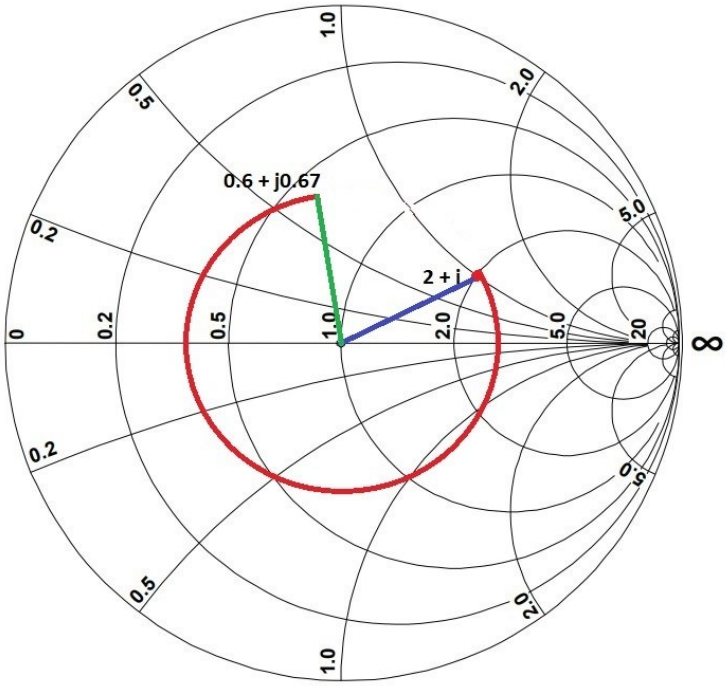
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Finding the load impedance seen by the transmitter

Now, let's take our plot a step further, and determine the load impedance as seen by the generator (transmitter), which might be a surprising concept for some. In other words, the transmission line (coax or other feed line) actually *transforms the load impedance* (makes it appear differently) along the transmission line. Let's say that the electrical (that is, taking velocity factor into account) length of our coax is 96 feet long, and we want to use it on 80 meters. A half-wavelength at, say, 3.92 MHz is 468 ÷ 3.92 MHz = 119.4 feet, and our feed line is 96 ÷ 119.4 = 0.8 of that half-wavelength (in other words, 0.4λ, or 0.4 times the whole wavelength).

To locate the new impedance point on the Smith chart, place the pivot point of a drawing compass at the Prime Center and the drawing point at **2 + j**, our red dot. Then, drawing an arc (red) clockwise (because we're drawing toward the generator) **0.8 x 360˚ = 288˚**, the compass drawing point ends at the impedance value seen by the generator, or about **0.6 + j0.67**:



We then un-normalize the value by multiplying by the characteristic impedance and get

$$(0.6 + j0.67) \times 50\ \Omega = 30 + j33.5\ \Omega$$

So, the load impedance as seen from the transmitter will be **30 + j33.5 Ω**. And because the imaginary (j) component is positive, the generator will see an *inductive load*. But wait, if our coax was only, say, 50 feet long, are we saying the transmitter will see a capacitive load? Yes, meaning we can change the antenna appearance by simply changing the coax length.

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Finding the SWR

By the way, the red arc we've added to the chart is part of the *SWR circle*, so that where it crosses the resistance axis between 1.0 and infinity is the load SWR. In our case, the SWR appears to be about **2.6:1** if we examined it on a more detailed chart. In some Smith charts, some whole-number constant-SWR circles are added to help you more easily see the load SWR (E9GØ9), and indicates that *the SWR is constant throughout the transmission line*, assuming a lossless line. If we had included the loss, the circle would appear as a spiral, spiraling slightly inward in the clockwise direction, because the load SWR appears less lossy to the transmitter.

Let's check our work. To calculate the SWR of a load, such as an antenna, the calculations derive from the reflection coefficient **Γ** (uppercase gamma), such that

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ and } SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Then, the actual SWR of an antenna (load) that exhibits $\mathbf{Z_L}$ **= 100 + j50** ohms impedance examined on a $\mathbf{Z_0}$ **= 50-ohm** coax would be

$$\Gamma = \frac{100 + j50 - 50\ \Omega}{100 + j50 + 50\ \Omega} = 0.4 + j0.2 \text{ and } |\Gamma| = \sqrt{0.4^2 + 0.2^2} = 0.4472$$

$$SWR = \frac{1 + 0.4472}{1 - 0.4472} = 2.618\ \checkmark$$

So, knowing the antenna impedance, we can manually calculate its SWR like we just did, or simply plot it on the Smith chart, draw an arc to the resistance axis, and immediately see the SWR right on the chart.

Finding the admittance

There are times when knowing the admittance, or the inverse of impedance, is convenient. This is especially true when calculating the impedance of parallel circuits (antenna trap or tank circuit), which can get really messy really fast, because of all the complex number calculations. In the case of our example, for an impedance of **Z = 100 + j50 ohms**, the corresponding admittance **Y** is the reciprocal of impedance, or

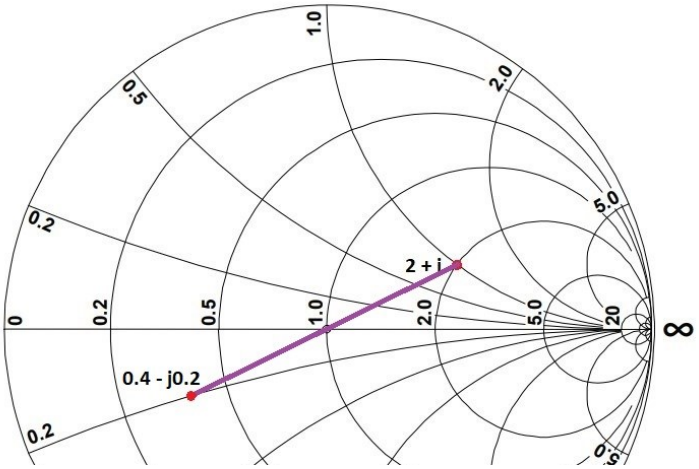
$$Y = \frac{1}{100 + j50\ \Omega}\left(\frac{100 - j50}{100 - j50}\right) = \left(\frac{100 - j50}{12500}\right) = 0.008 - j0.004\ \Omega^{-1}$$

Now, let's see whether we can use a Smith chart to verify our calculation. Instead of drawing a line from the Prime Center to our impedance point **2 + j**, draw it in the opposite direction, and for the same length. This ends up at about **0.4 – j0.2** on the chart:

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Then, to convert it to admittance, divide by 50 ohms:

$$Y = \frac{1}{0.4 - j0.2}\left(\frac{1}{50\ \Omega}\right) = 0.008 - j0.004\ \Omega^{-1}\ \checkmark$$

Working backwards – a real-life example

Alright, let's say you have an antenna on the roof, connected by 75 feet of RG-8X coax coming into your shack. On the shack end of the coax, your analyzer on 40 meters (say 7.2 MHz) shows an impedance of **31 – J5.5 ohms**. What is the impedance of your antenna on 40 meters?

First, let's find the chart plot for the impedance on the shack end of the coax. We normalize our shack-end impedance by dividing by 50 ohms, the coax characteristic impedance:

$$\frac{31 - j5.5\ \Omega}{50\ \Omega} = 0.62 - j0.11$$

Next, let's find how far around the circle we need to draw, to locate our antenna impedance. The length of our coax is 75 feet long, but we're interested in its electrical length, which is what's reflected on the Smith chart. Let's use the half-wave factor of **468** to convert a half-wavelength of ***ideal*** coax into feet, then multiply by the **RG-8X** coax velocity factor of 0.82:

$$\frac{468}{7.2\ \text{MHz}} = 65\ \text{ft, and } 65\ \text{ft} \times 0.82 = 53.3\ \text{ft}$$

We need to find how long our 75-foot coax is, in numbers of half-wavelengths, then truncate the whole number of multiple half-wavelengths:

$$\frac{75\ \text{ft}}{53.3\ \text{ft}} = 1.407\ \text{half-wavelengths, truncated} = 0.407\ \text{half-wavelengths}$$

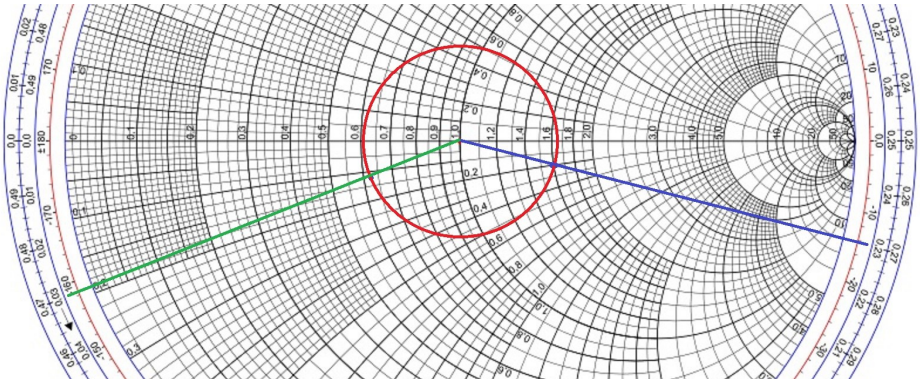
Plotting the shack impedance on the chart as **0.62 – j0.11** matches up to about **–158.5°** on the

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outer ring (green line). Since an entire half-wavelength is **360**˚ around the outer ring, **0.407** half-wavelengths is **0.407 x 360**˚ **= 146.5**˚. Therefore, our antenna impedance will be located **146.5**˚ counter-clockwise (because we're going from the shack to the antenna) direction around the outer ring, or **–158.5**˚ **+ 146.5**˚ **= –12**˚ (blue line). We now plot the new point the same distance from the Prime Center (red circle, with our compass) and arrive at **1.6 – j0.2**:



Multiplying the new point by 50 ohms gives the load (antenna) impedance:

$$Z_L = (1.6 - j0.2) \times 50\ \Omega = 80 - j10\ \Omega$$

And by the way, as you can see, the antenna's SWR is about 1.64:1.

Finally

The Smith chart has uses far beyond what I've described here, such as designing matching circuits and tuning stubs (E9GØ5). But in order to keep this article to a reasonable size, I've only introduced you to the *fundamentals* of how to use one. I have also not explained the horizontal scale below the circular graph, but much of that is redundant with what I've already shown you. For a more in-depth look at Smith charts, check out the tutorial page on ARRL.

Plenty of online and desktop software easily handle the calculations that were once done using a Smith chart. While it remains a quick graphical description of impedance behavior on a transmission line to those who are accustomed to seeing it, most engineering work that was once performed using a Smith chart is now made much simpler by the apps. Furthermore, some of the software, as well as VNAs and antenna analyzers, also display Smith charts.

Summary

The Smith chart is a graphical tool used for solving problems involving transmission lines and matching circuits, as well as other RF engineering concerns. It's made from resistance circles and orthogonal reactance arcs, all originating from a point on the right side of the diagram. With it, one can plot the antenna impedance, then determine SWR, admittance, and the impedance as seen by the transmitter far down the coax. Today, the Smith chart is largely replaced by software for practical work, but remains a point of historical curiosity for many.

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